Technical Notes

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Mass Flux Boundary Conditions in Linear Theory

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Introduction

THE use of an approximate mass flux boundary condition in place of the "exact" velocity boundary condition has been proposed by numerous investigators for the computation of the linearized flow over aerodynamic bodies (Refs. 1-3 provide several examples). In fact, it is the recommended boundary condition in the widely used PANAIR program.[4] However, the results using the mass flux condition are not necessarily better and they are often worse than solutions obtained using the velocity boundary condition. This is particularly true at supersonic speeds. In this Note, a perturbation analysis is used to demonstrate that the commonly used mass flux boundary condition is inconsistent in that it neglects a term that is of the same order in thickness ratio ϵ as the extra term included in the mass flux boundary conditions (BC) proposed in Refs. 1-3. When the consistent mass flux BC is used, the results using either the "exact" velocity or mass flux BCs are shown to be essentially equivalent.

Perturbation Analysis

The boundary condition is formed by writing the mass flux and normal vectors, combining and then dropping the higher-order terms. Using linear theory scaling, the density and velocity are given by

$$\rho=\rho_\infty(1-\epsilon M_\infty^2 u)+\mathcal{O}(\epsilon^2)$$

and

$$\hat{q}=U_\infty[(1+\epsilon u)\hat{i}+\epsilon v\hat{j}]$$

so that

$$\rho\hat{q}=\rho_\infty U_\infty\{[1+\epsilon\beta^2 u+\mathcal{O}(\epsilon^2)]\hat{i} + \epsilon v[1-\underline{\epsilon M_\infty^2 u}+\mathcal{O}(\epsilon^2)]\hat{j}\} \tag{1}$$

where ϵ is a small parameter taken to be the order of the thickness ratio of the body. The underlined term has been neglected previously on the grounds that it is second order in ϵ. However, as shown here, this term leads to a first-order contribution that must be retained in the boundary conditions to obtain a consistent formulation. For the surface given by

$$y=\epsilon f(x)$$

the unit normal vector is

$$\hat{n}=\frac{1}{\sqrt{1+\epsilon^2 f'^2}}[\epsilon f'(x)\hat{i}-\hat{j}] \tag{2}$$

or

$$\hat{n}=\epsilon n_x\hat{i}+n_y\hat{j}$$

(The failure to properly scale the n_x term with ϵ, as in this equation, was responsible for the derivation of the inconsistent form of the mass flux boundary condition in Ref. 3 and subsequently in Refs. 1 and 2.)

The mass flux BC is then found by

$$\rho\hat{q}\cdot\hat{n}=0$$

so that

$$\rho\hat{q}\cdot\hat{n}=\rho_\infty U_\infty[\epsilon(1+\epsilon\beta^2 u)n_x+\epsilon(1-\epsilon M_\infty^2 u)n_y v]+\mathcal{O}(\epsilon^3)$$

becomes

$$(1+\epsilon\beta^2 u)n_x+(1-\underline{\epsilon M_\infty^2 u})n_y v=\mathcal{O}(\epsilon^2) \tag{3}$$

The underlined term is the one omitted previously. The usual linearized boundary condition follows from the neglect of the two $\mathcal{O}(\epsilon)$ terms in Eq. (3), while a consistent second-order condition follows from the retention of both these terms. There is no theoretical justification for neglecting just one of these terms while retaining the other.

The consistent mass flux BC is obtained by retaining $\mathcal{O}(\epsilon^2)$ terms in the mass flux relation, forming the BC, and then retaining the first-order terms in the final expression. Simplifying the mass flux relation before forming the BC leads to an inconsistent result.

An Example: Supersonic Flow over a Cone

The results obtained above are illustrated for the supersonic flow over a cone in Fig. 1. This case was used to illustrate the inconsistent mass flux result previously.[1] The "exact" velocity BC is

$$\tan\delta_c=\epsilon v/(1+\epsilon u) \tag{4}$$

and the mass flux BC is

$$\tan\delta_c=\epsilon\frac{(1-\underline{\epsilon M_\infty^2 u})v}{(1+\epsilon\beta^2 u)}+\mathcal{O}(\epsilon^3) \tag{5}$$

The inconsistent mass flux BC is obtained by dropping the underlined term in the numerator to give

$$\tan\delta_c=\epsilon\frac{V}{1+\epsilon\beta^2 u} \tag{6}$$

It is clear that the omitted term is of the same order of magnitude as the additional term in the denominator. In fact, using the binomial theorem, Eq. (5) reduces to the exact BC [Eq. (4)] to $\mathcal{O}(\epsilon^3)$. The results in Fig. 1 compare the linearized solution obtained with the three forms of boundary conditions to the exact inviscid solution for supersonic flow over a 15 deg cone as a function of freestream Mach number. The linearized pressures were computed from the velocities by applying the boundary condition on the surface and using the standard slender-body pressure coefficient formula, which is identical to the procedure used in obtaining these same results in Ref. 1 for the inconsistent BC. The results indicate that the consistent form of the mass flux boundary condition gives

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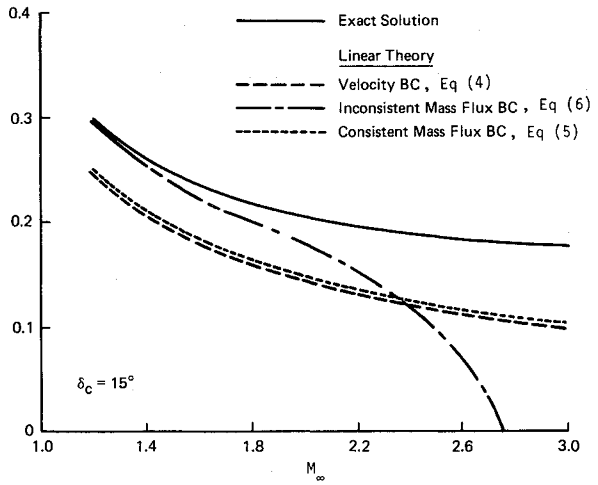

Fig. 1 Comparison of surface pressures on a cone for the various boundary condition relations.

results that are nearly identical to those obtained with the exact velocity boundary condition [Eq. (4)]. The difference in agreement with changing Mach number for the inconsistent mass flux BC is typical of results obtained using equations that have been derived without insuring a uniform order of magnitude of all terms. It also demonstrates vividly the M_∞^2 dependence of the heretofore neglected term.

The difference between the exact solution and the linearized solutions using consistent boundary conditions [i.e., either Eq. (5) or (6)] is clearly due to approximations in the differential equation and the formula used for the pressure coefficient. The close agreement achieved with the inconsistent boundary condition at low Mach numbers is spurious and cannot be relied on as an argument in favor of the inconsistent mass flux boundary condition.

References

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Application of Photon Correlation to Turbulent Fluid Mechanics

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Introduction

WITHIN the past few years, the commercial availability of fast digital correlators has led to new techniques of measurement based upon the quantum resolved properties of low-light levels. The photon correlation processing technique used in laser velocimetry applications has remarkable power in that it covers a wide dynamic range, is sensitive to extremely low scattered light intensities, and does not require a continuous signal. The low-light level requirements permit the use of relatively small lasers and naturally occurring contaminants as the scattering centers. However, there are drawbacks to the photon resolved technique, such as sensitivity to background ambient light and flare, photon pileup, modest control over the size distribution of the scattering centers, and the facts that, only ensemble-averaged data are retrievable, and mean velocity and turbulent intensity information alone are calculable from the correlation function.

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Experimental Design

In order to establish the credibility of the photon correlation technique, a series of different fluid flow configurations is devised and documented. The first segment of the experimental investigation focuses on the measurement of a rectangular nozzle turbulent jet.[1] Considerable data obtained via hot wire anemometry exist for the downstream flow. In addition, the exit plane mean velocity can be varied up to a maximum speed of 260 m/s, which is in the compressible regime. The naturally occurring contaminant present in the laboratory-compressed air supply is used as the source of marking particles.

The naturally "seeded" atmosphere furnished sufficient scattering centers to also allow the photon correlation scheme to be employed in the experimental investigation of the inlet duct flow occurring prior to the combustion chamber of a jet engine.[2] The engine inlet is placed outside the walls of the laboratory. Mean velocities and turbulent intensities are obtained at three different locations, namely: 1) immediately downstream of the leading edge of the bypass contraction; 2) immediately downstream of the venturi section throat; and 3) immediately downstream of the exit from the bypass diffuser. Two different throttle settings (i.e., 50 and 70%) are used.

Difficulties in using a scattering based laser velocimeter led to the next portion of the investigation – the measurement of the velocity field behind a stationary flat disk immersed in and normal to the direction of a subsonic turbulent jet.[3] Once again mean and fluctuating information is sought.

A quanititative analysis of the sensitivity of the photon correlation scheme to incident laser intensity is the next goal of this research. The approach taken is to vary the actual size of the laser beam by means of beam expander/aperture arrangement. The $1/e^2$ beam diameter is varied with the resultant different sized control volumes located in flowfield of the turbulent jet described previously.

With the application of the photon technique to an investigation of a flowfield located in the test section of a wind tunnel, the presence of optical plates and confining walls and their resultant effects on signal quality is noted. Problems associated with flare and/or background light are found to exist, and create uncertainty with respect to both mean and turbulent velocity data. The flowfield chosen for this portion of the project is a two-dimensional wake.[4]

A flapping (oscillating) jet allowed the sensitivity of the photon correlation scheme to an unsteady flowfield to be documented. The nozzle design employed consists of a modified fluidic element with a feedback mechanism.[5] A centrifugal blower with naturally occurring contaminant serving as scatters is used.

Experimental Results and Discussion

Rectangular Nozzle Jet

Mean velocity profiles taken by the photon correlation technique and hot wire anemometry are shown in Fig. 1. Here, the nondimensionalized velocity ratio, U_{CEN}/U_{CORE}, is plotted vs downstream distance, X, from the exit plane. The quantity U_{CORE} is the exit plane mean velocity. Three dif-